A damped harmonic oscillator model for the Madden-Julian Oscillation

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E. C. J. Oliver[1,2] and K. R. Thompson[2]

[1] Institute for Marine and Antarctic Studies, University of Tasmania, Hobart, Australia
[2] Department of Oceanography, Dalhousie University, Halifax, Canada

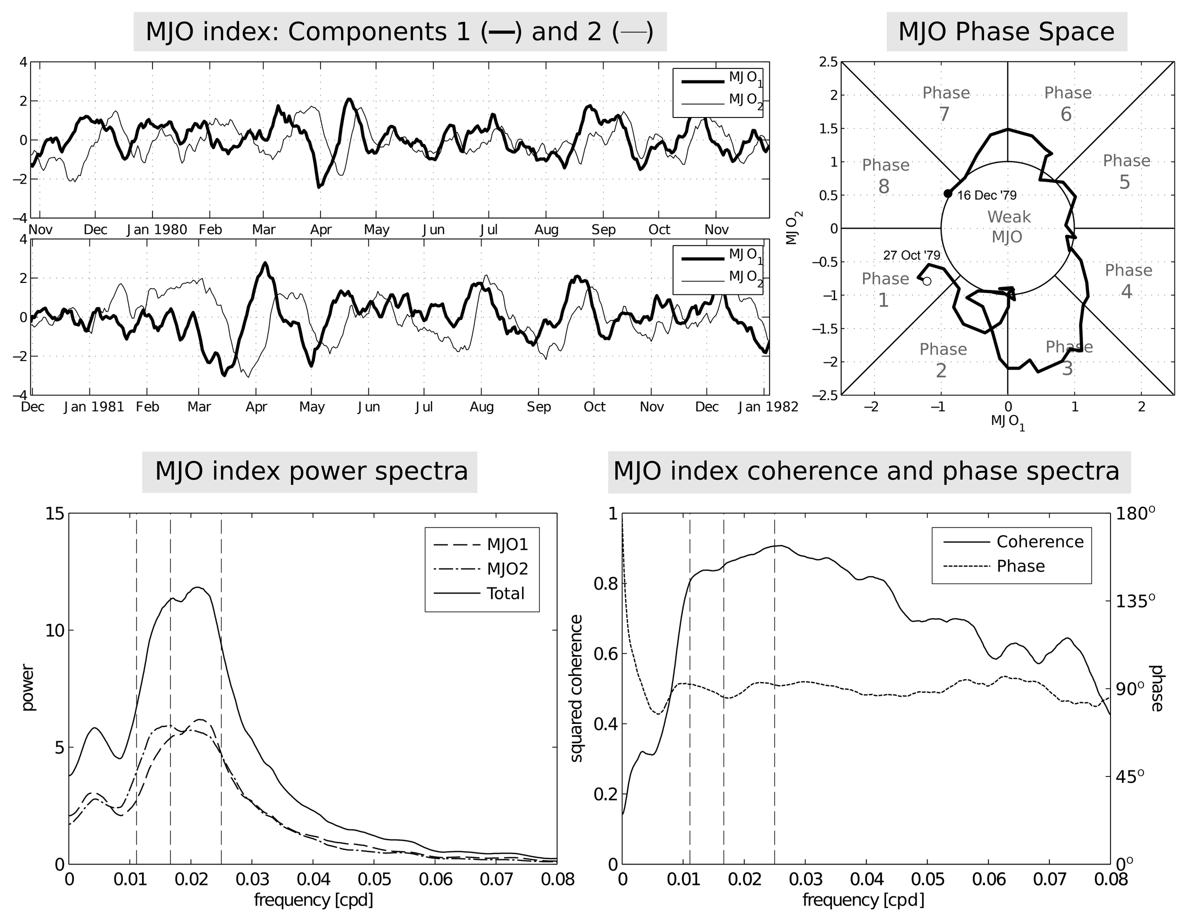
IMAS Institute for Marine and Antarctic Studies · GOAPP Global Ocean-Atmosphere Prediction and Predictability · UTAS

1 Introduction

- The atmosphere is a chaotic system, implying that there exists a **time scale** beyond which **predictions** based on similar initial conditions will evolve into considerably different states.
- Lorenz (1965) estimated the limiting time scale for **weather prediction** to be about one to two weeks.
- The **Madden-Julian Oscillation (MJO)** is the dominant mode of intraseasonal variability (40-60 days) in the tropical atmosphere, with strong features in precipitation, cloud cover, and zonal wind and upper (850 mbar) and lower (200 mbar) levels.
- The **MJO has a time scale of predictability** beyond the synoptic "two-week limit" of weather variability, estimated to be in the range of 10 to 30 days.

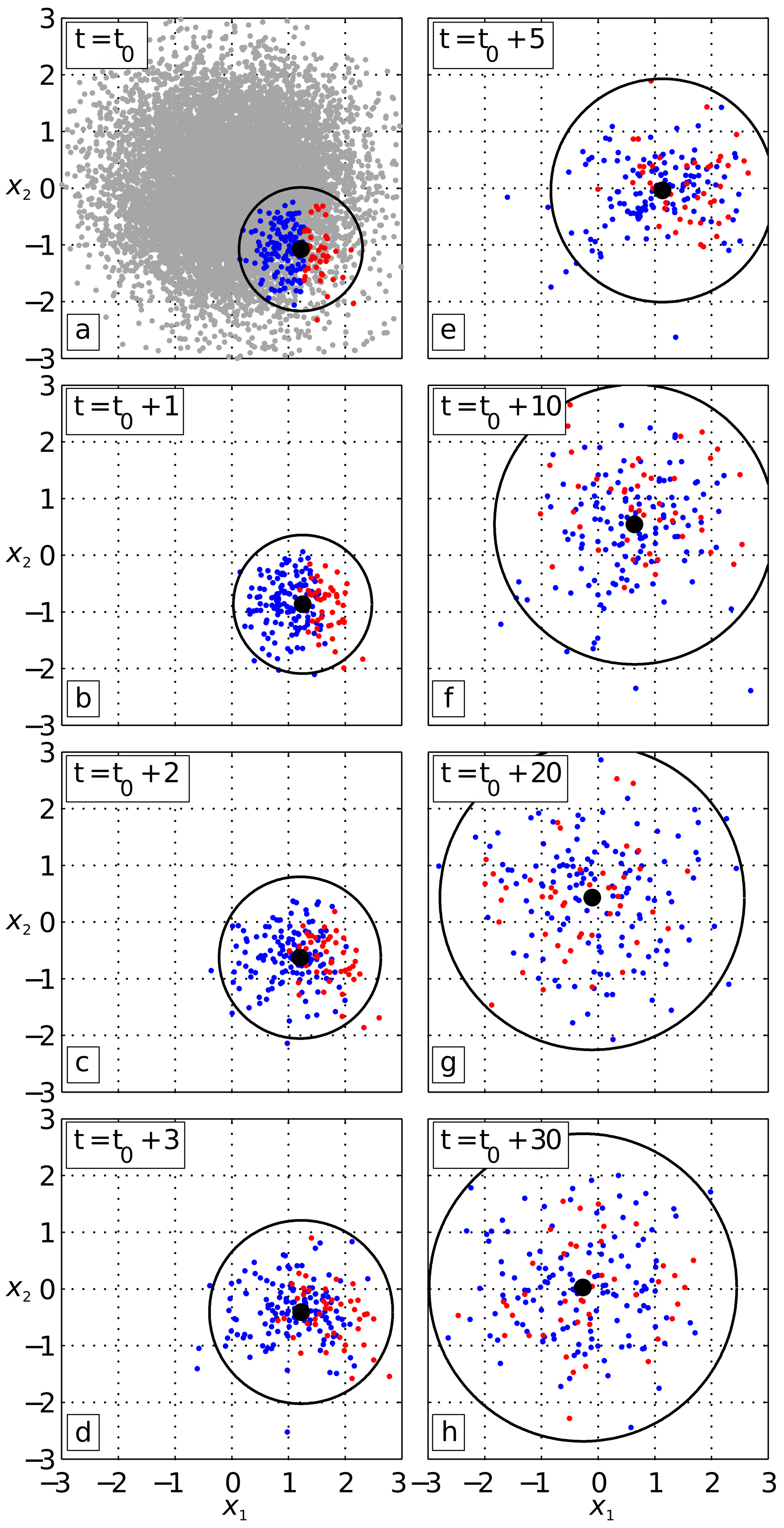
2 The Madden-Julian Oscillation Index

- The most widely accepted **characterization of the MJO** is the index developed by Wheeler and Hendon (2004).
- This index, consisting of **two oscillatory time series**, is based on satellite-based measurements of outgoing longwave radiation and reanalysis representations of zonal wind.
- The majority of energy lies in a band of oscillation periods between 40 and 60 days, and the two components are **highly coherent** over this band (0.8-0.9) and are **90^o out of phase**.
- When the two index components are plotted against each other they form an "**MJO phase space**", within which MJO events move in a counterclockwise manner as they propagate eastwards.



3 MJO Event Behaviour

- The **distribution of all MJO index values** are shown below as grey dots and a subset (an ensemble; centred on black dot with 95% enclosure shown by the circle) are shown as red and blue dots
- The **ensemble members are tracked** for the following 30 days, with the ensemble mean and variance shown by the black dot and circle



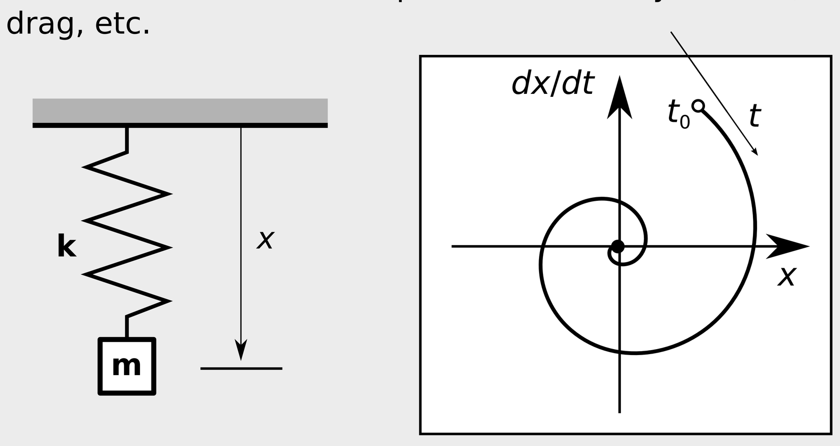
- The ensemble mean exhibits **rotation** and **decay**
- The ensemble variance increases with time asymptotically to the MJO index variance
- The within-ensemble correlation decays with time
- Behaviour is reminiscent of a **damped harmonic oscillator**

Aside: Damped harmonic oscillator

A harmonic oscillator is a linear system in which perturbations are returned to the equilibrium state by a force proportional to the perturbation distance, leading to oscillatory motion (think: a mass on a spring)

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

A damped harmonic oscillator has the further property that oscillations are damped over time by friction, air drag, etc.



4 Oscillator Model

- A damped harmonic oscillator can be expressed in discrete time as a **bivariate autoregressive (AR[1]) process**:

$$\boldsymbol{x}_{t+1} = \boldsymbol{A}_1 \boldsymbol{x}_t + \boldsymbol{f}_{t+1}$$

where $\boldsymbol{x} = [x_1\ x_2]^T$ represents the oscillator components (akin to position and velocity, or the two MJO index components). Each time step the system is **decayed by the factor γ_1** and **rotated through an angle θ** by the state matrix $\boldsymbol{A}_1$,

$$\boldsymbol{A}_1 = \gamma_1 \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \quad \boldsymbol{A}_2 = \gamma_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the system is forced by autoregressive forcing $\boldsymbol{f}_t$:

$$\boldsymbol{f}_{t+1} = \boldsymbol{A}_2 \boldsymbol{f}_t + \boldsymbol{\epsilon}_{t+1}$$

where $\boldsymbol{\epsilon}_t$ is white noise forcing with covariance $\boldsymbol{\Sigma}_\epsilon = \sigma_\epsilon^2 \mathrm{I}$

- Mean:

$$\boldsymbol{\mu}_{t_0+k} = \boldsymbol{A}^k \boldsymbol{\mu}_{t_0}$$

- Covariance:

$$\boldsymbol{\Sigma}_{t_0+k,t_0} = \boldsymbol{A}^k \boldsymbol{\Sigma}_{t_0,t_0}$$

- Correlation:

$$\rho_k^2 = \frac{\mathrm{tr}(\boldsymbol{\Sigma}_{t_0+k,t_0} \boldsymbol{\Sigma}_{t_0,t_0}^{-1} \boldsymbol{\Sigma}_{t_0,t_0+k})}{\mathrm{tr}(\boldsymbol{\Sigma}_{t_0+k,t_0+k})}$$

Re-expressing as a quadrivariate AR(1) model, with state matrix $\boldsymbol{A}$, the evolution of the model mean, covariance, and correlation from an initial state at time t_0 are straightforward to calculate

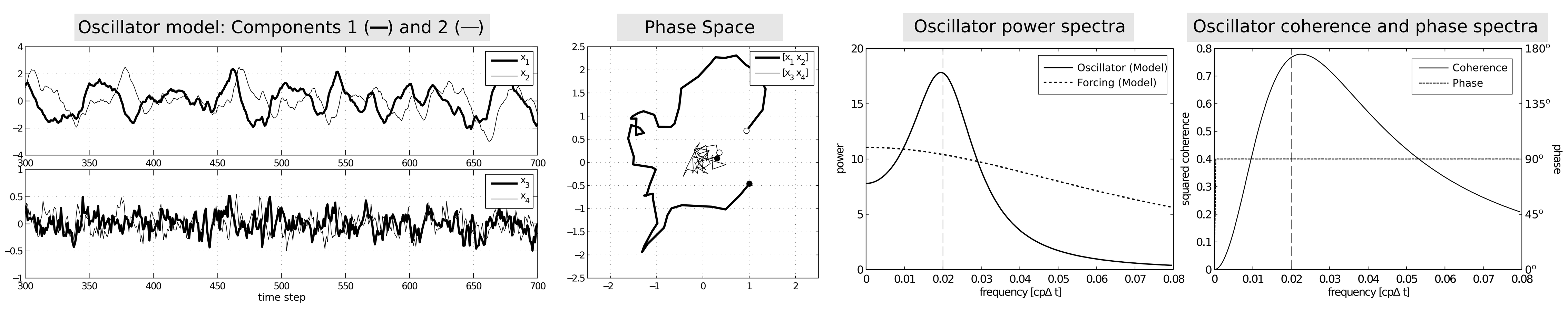
$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 & \mathrm{I} \\ \boldsymbol{0} & \boldsymbol{A}_2 \end{bmatrix}$$

- The parameters are expressed as **timescales**, in units of days:

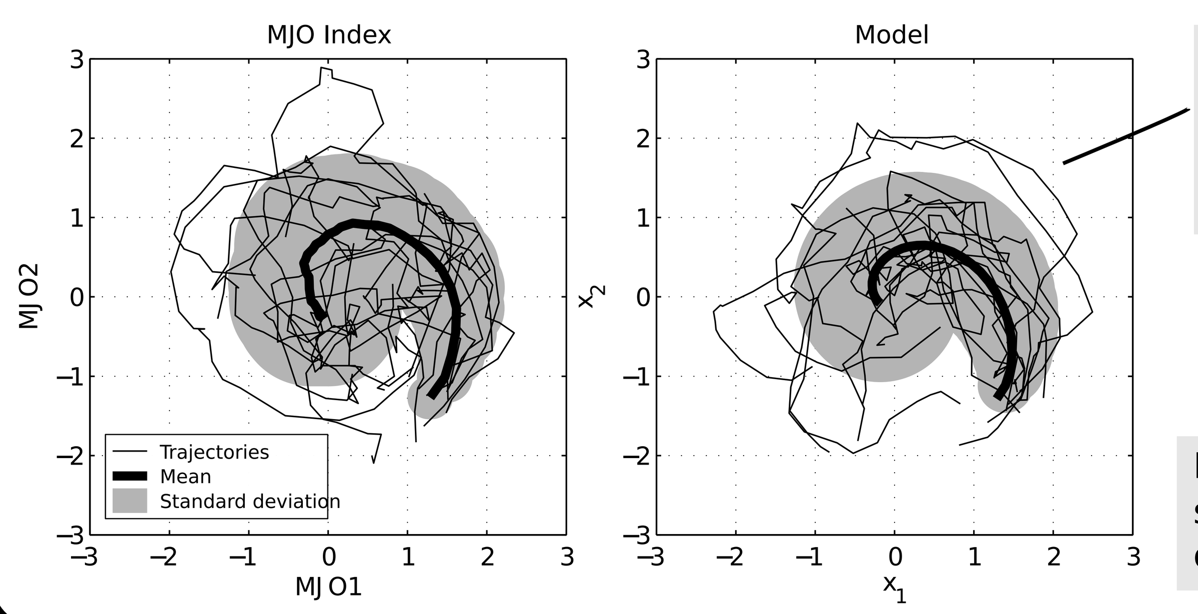
$$\tau_1 = -1/\log\gamma_1, \quad \tau_2 = -1/\log\gamma_2, \quad P = 2\pi/\theta$$

5 Model Fit and Predictability Measures

- Model **reproduces well** the time series and its spectral properties (**τ_1 = 15 days, τ_2 = 2 days, P = 50 days**)



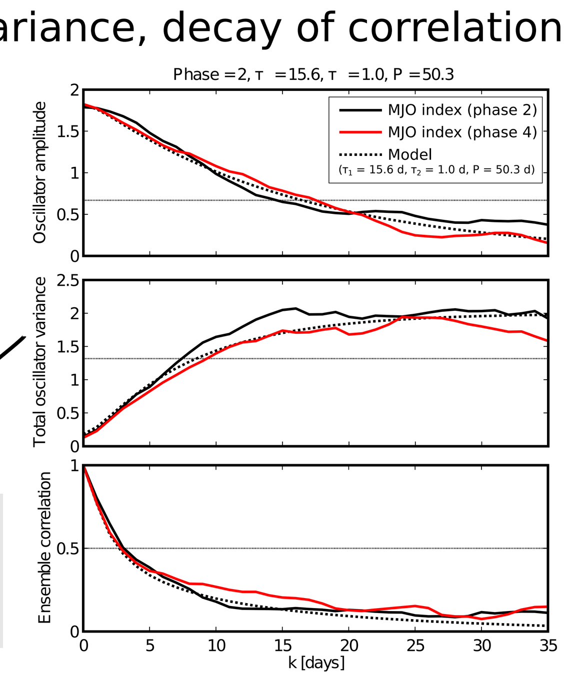
- Model also reproduces the ensemble behaviour: **rotation and decay**, increase of variance, decay of correlation



Ensemble statistics of observed index and model predictions, given initial condition in phase 3/4

Evolution of mean, variance, and correlation for phases 2 and 4

Define predictability time scales τ_μ, τ_{σ^2}, and τ_ρ based on thresholds (—)



References

- Lorenz (1965), *Tellus*, 17(3): 321-333
- Oliver and Thompson (2012), *Journal of Climate*, 25: 1996-2019
- Oliver and Thompson, A damped harmonic oscillator model for the Madden-Julian Oscillation, *in prep.*
- Wheeler and Hendon (2004), *Monthly Weather Review*, 132: 1917-1932

Conclusions 6

- Observed MJO index exhibits **rotation and decay**, behaviour reminiscent of a **damped harmonic oscillator** (DHO).
- A simple DHO **model captures** the temporal and spectral properties of the MJO index as well as its **basic predictability features**.
- The model parameters vary when fit as a function of initial condition, leading to the conclusions that: (i) a **more complex model is required** to fully capture MJO predictability, and (ii) **MJO predictability may vary** significantly with initial condition in MJO phase space.

Oscillator predictability as a function of initial condition in MJO phase space shows considerable variability, indicating that **perhaps the proposed model is insufficient**