

Predictability of the MJO index: Seasonality and phase-dependence

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- The **Madden-Julian Oscillation (MJO)** is dominant mode of intraseasonal (30-90 day) variability in the Tropics
- Expressed as
 - Deep convection, cloud cover
 - Rainfall
 - Low- and high-level winds
- Develops over Indian Ocean and propagates eastward, 5-10 m/s
- Influences generation of tropical cyclones, sea level variations, extratropical air temperature, etc...

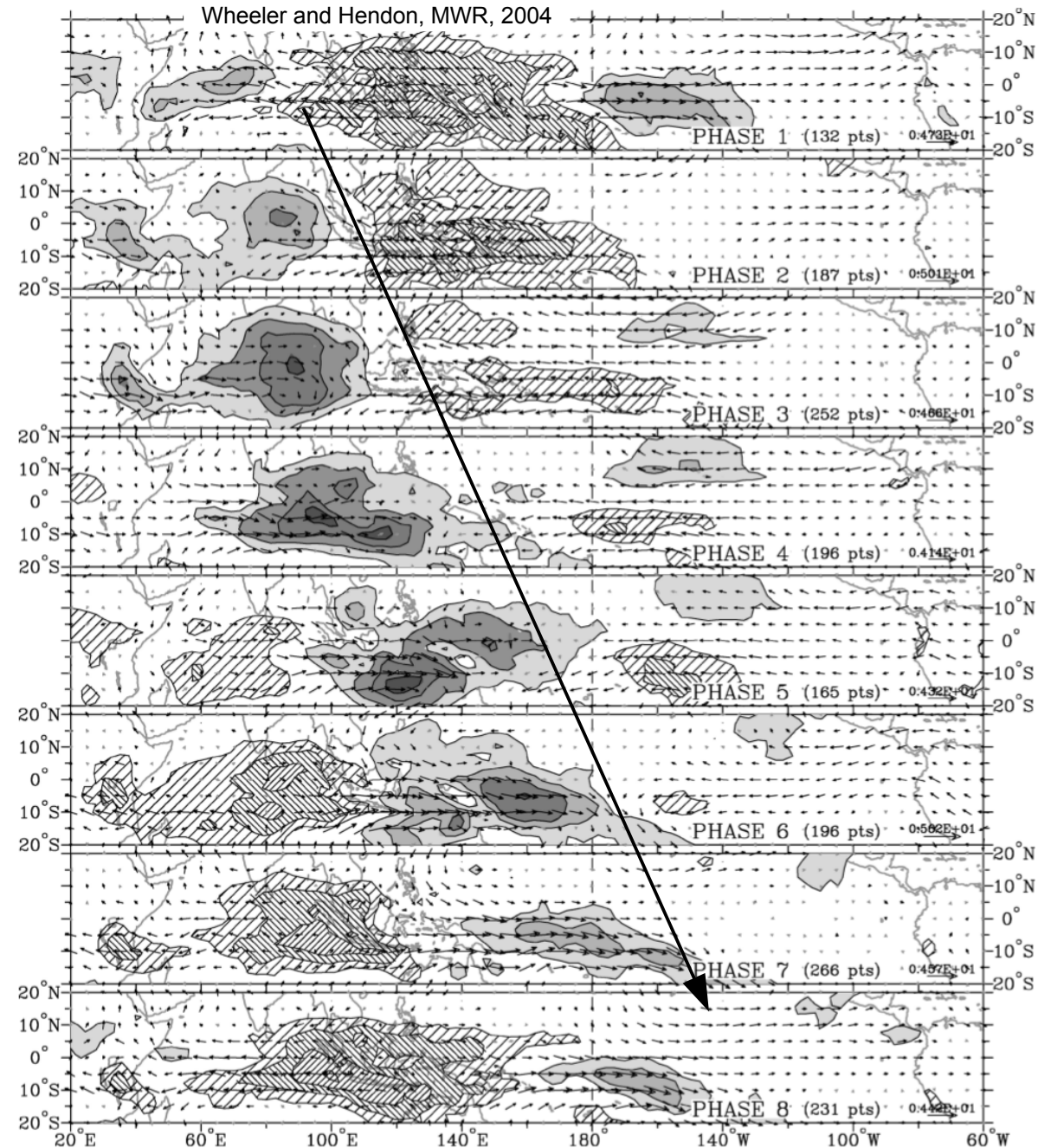
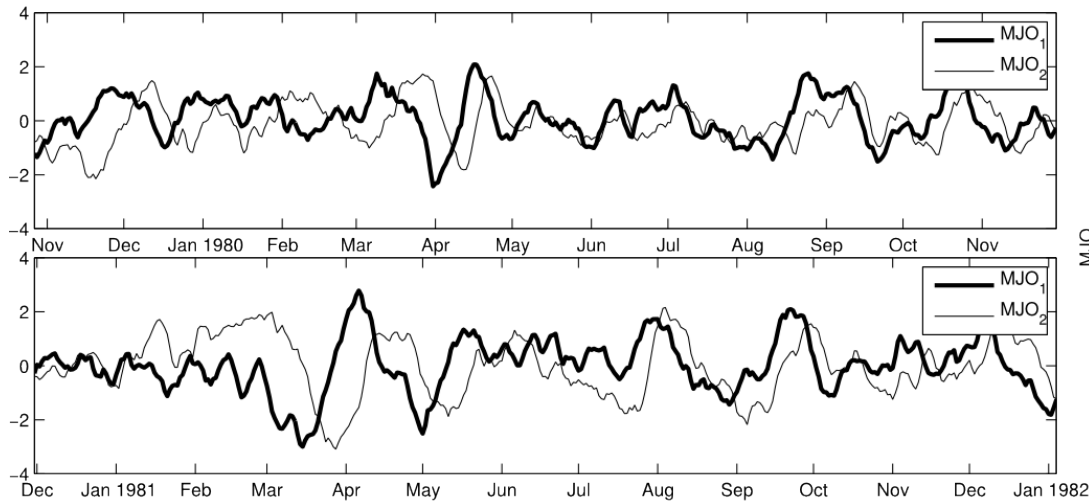


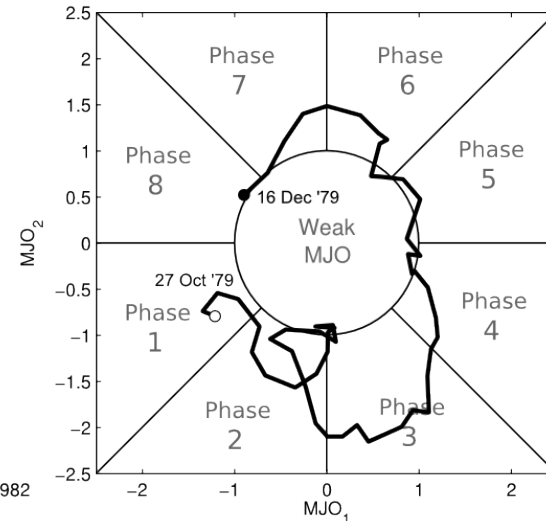
FIG. 8. DJF composite OLR⁴ and 850-hPa wind vector anomalies. Shading levels denote OLR anomalies less than -7.5 , -15 , -22.5 , and -30 W m^{-2} , respectively, and hatching levels denote OLR anomalies greater than 7.5 , 15 , and 22.5 W m^{-2} , respectively. Black arrows indicate wind anomalies that are statistically significant at the 99% level, based on their local standard deviation and the Student's t test. The magnitude of the largest vector is shown on the bottom right, and the number of days (points) falling within each phase category is given.

- Wheeler and Hendon (2004) (or WH04) **MJO index** based on 2 PCs of intraseasonal outgoing longwave radiation (OLR), and zonal wind at 200 hPa and 850 hPa:

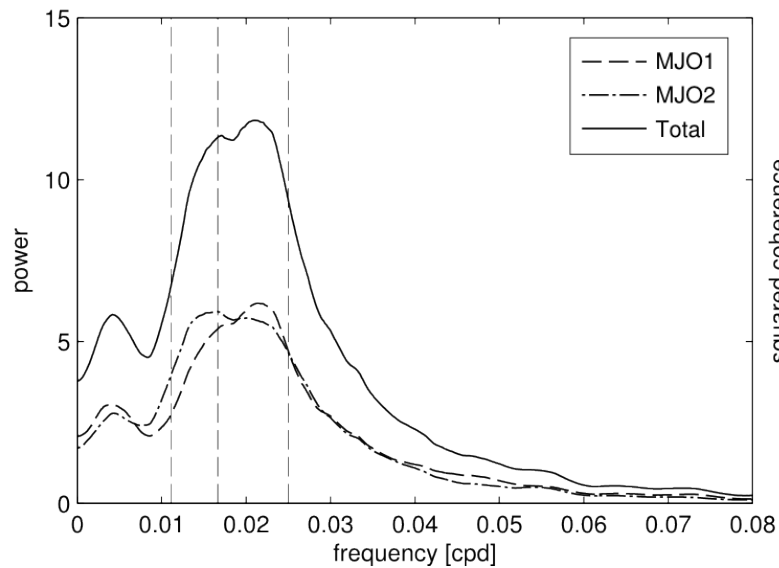
MJO index: Components 1 (—) and 2 (—)



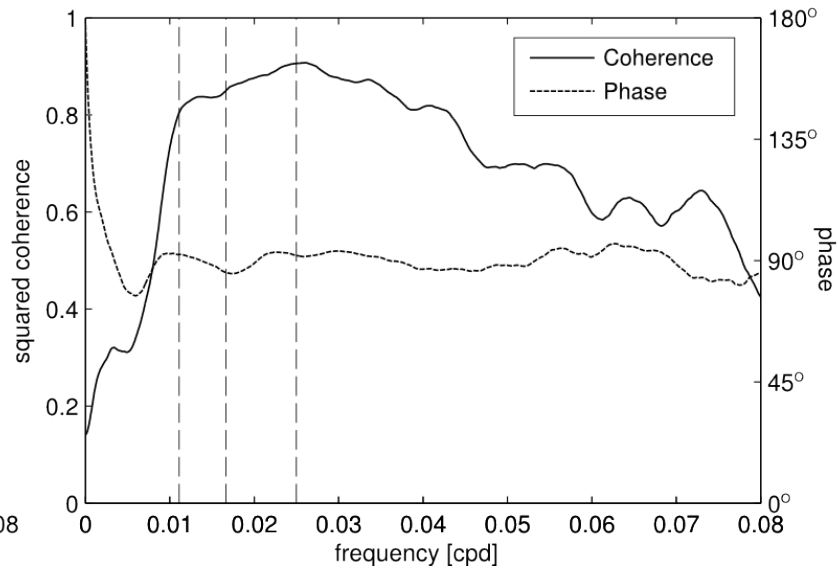
MJO Phase Space



MJO index power spectra

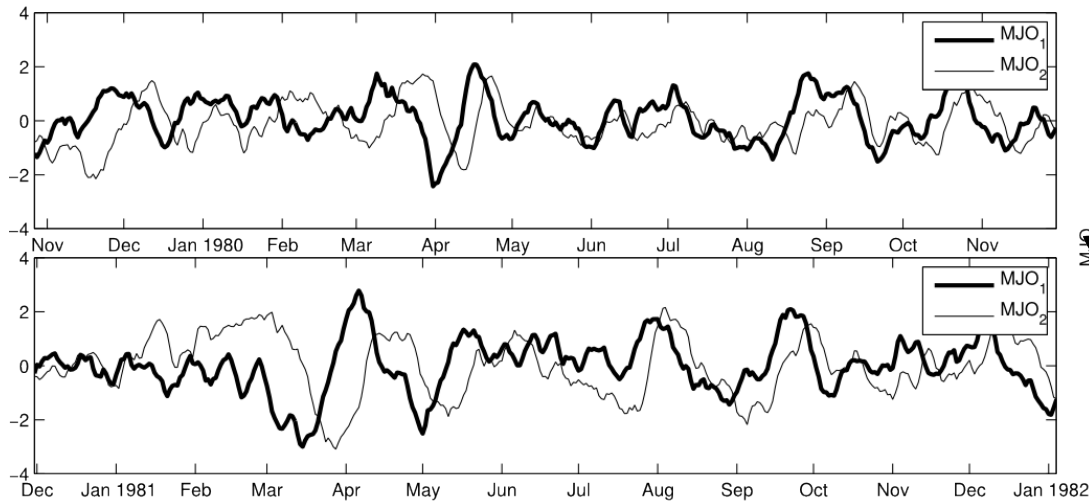


MJO index coherence and phase spectra

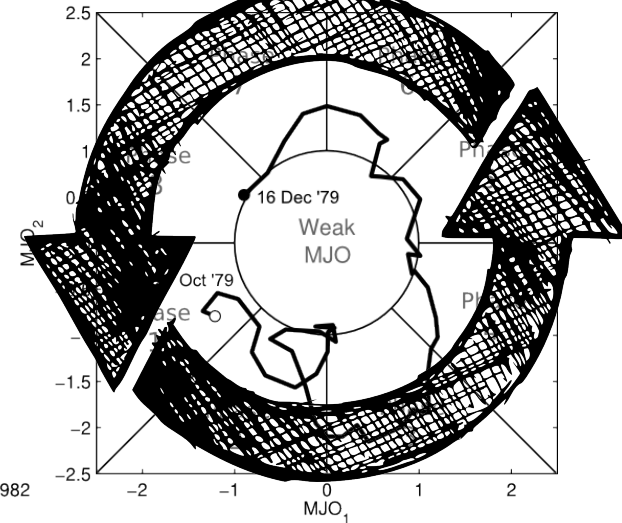


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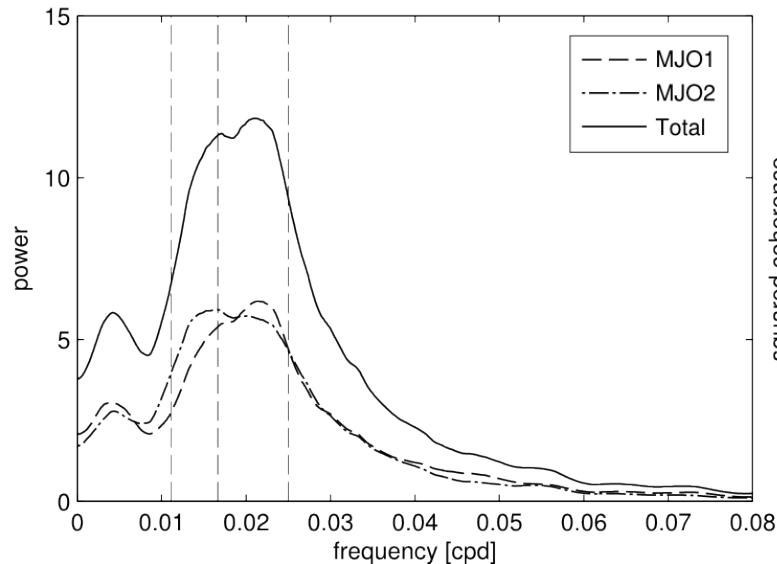
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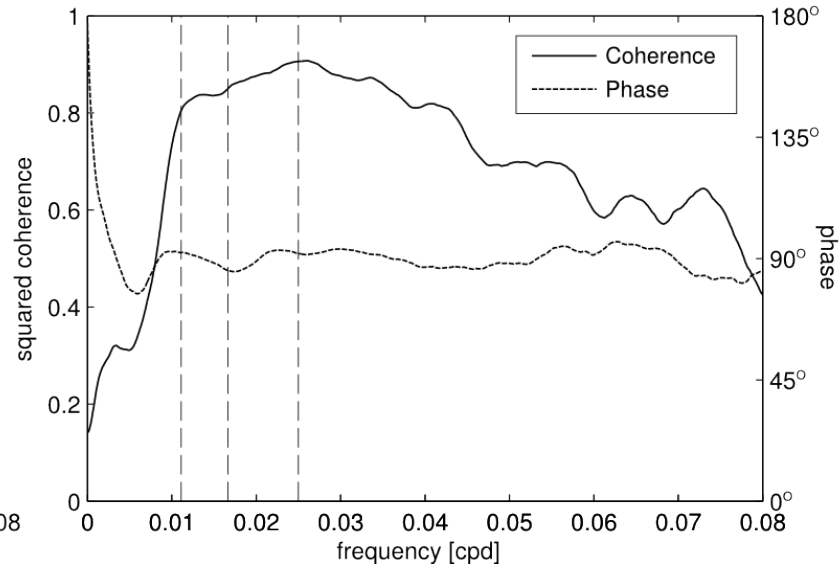
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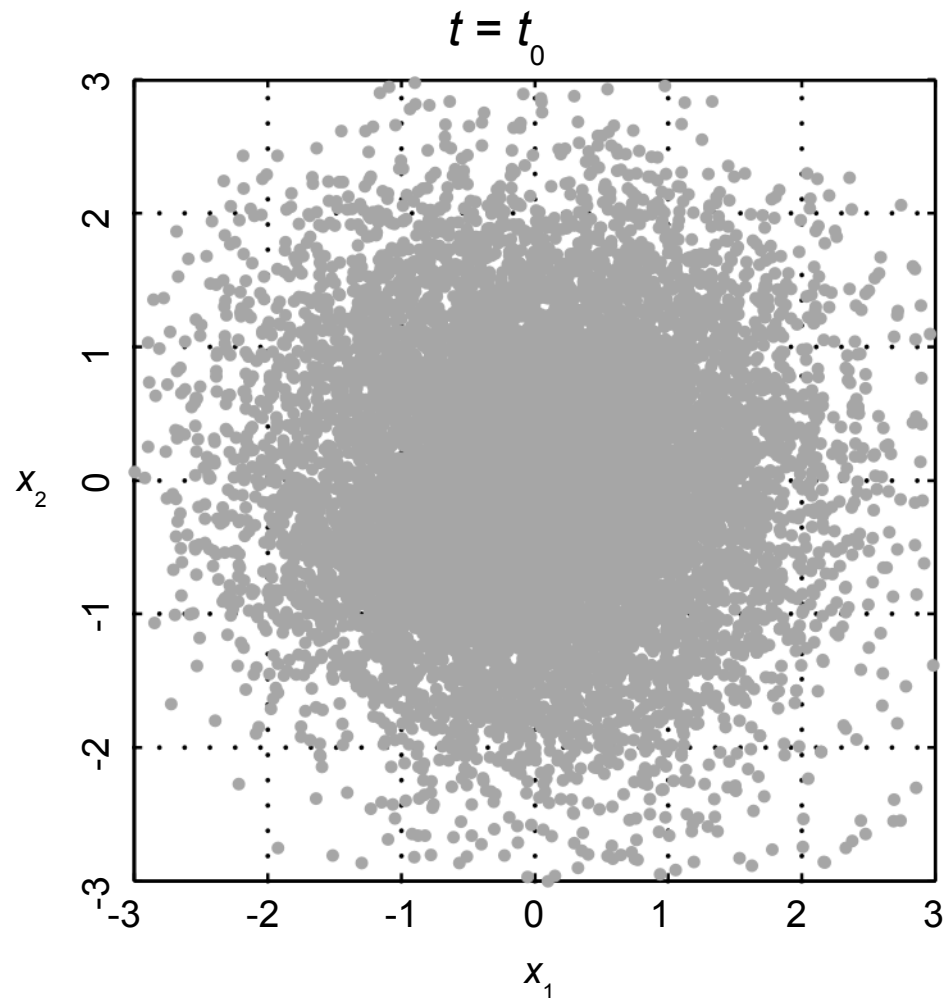
MJO index power spectra



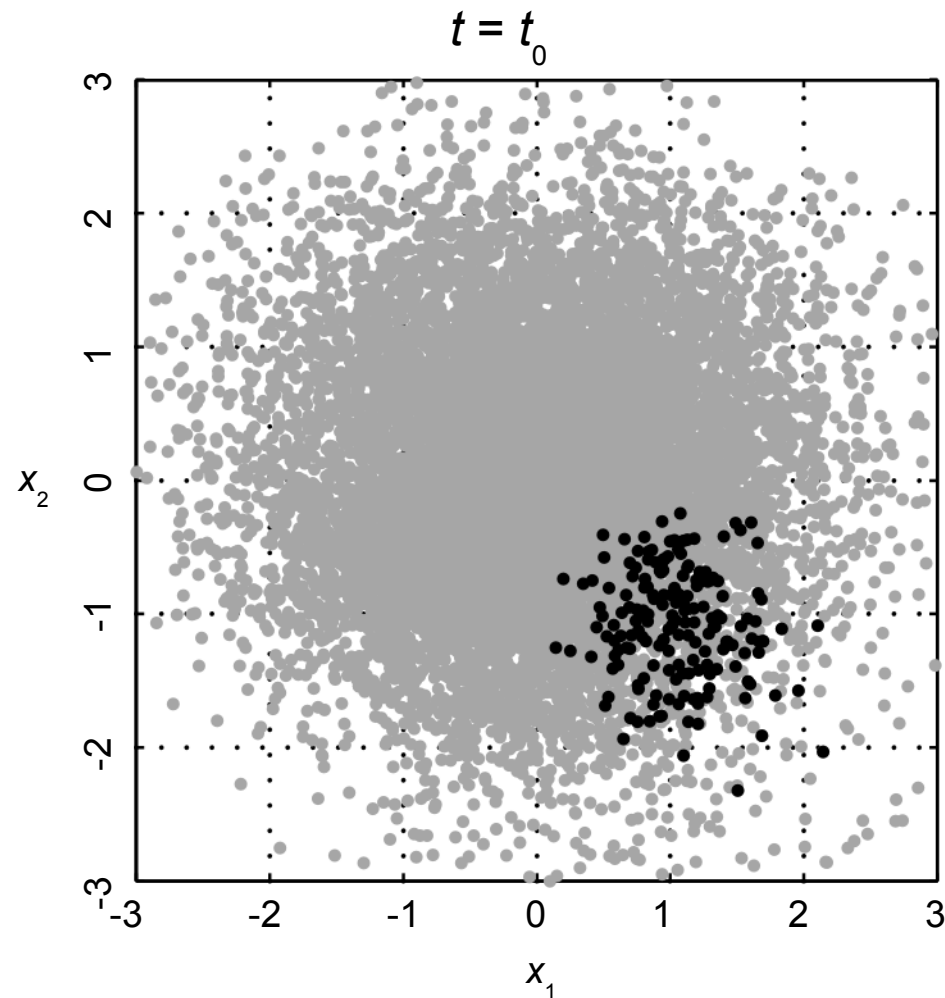
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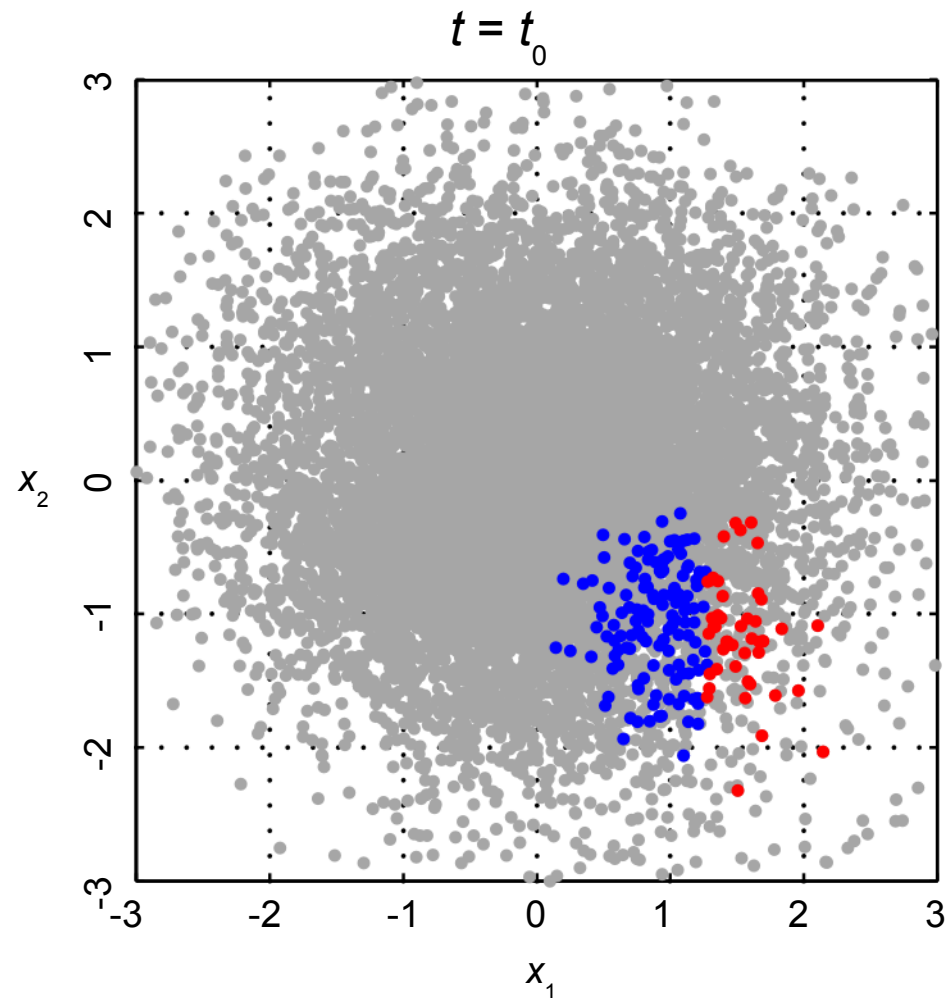
- Distribution of all MJO index values distributed approx. as bivariate normal (grey)
- A subset is chosen (red, blue) with mean (dot) and 95% enclosure (circle)



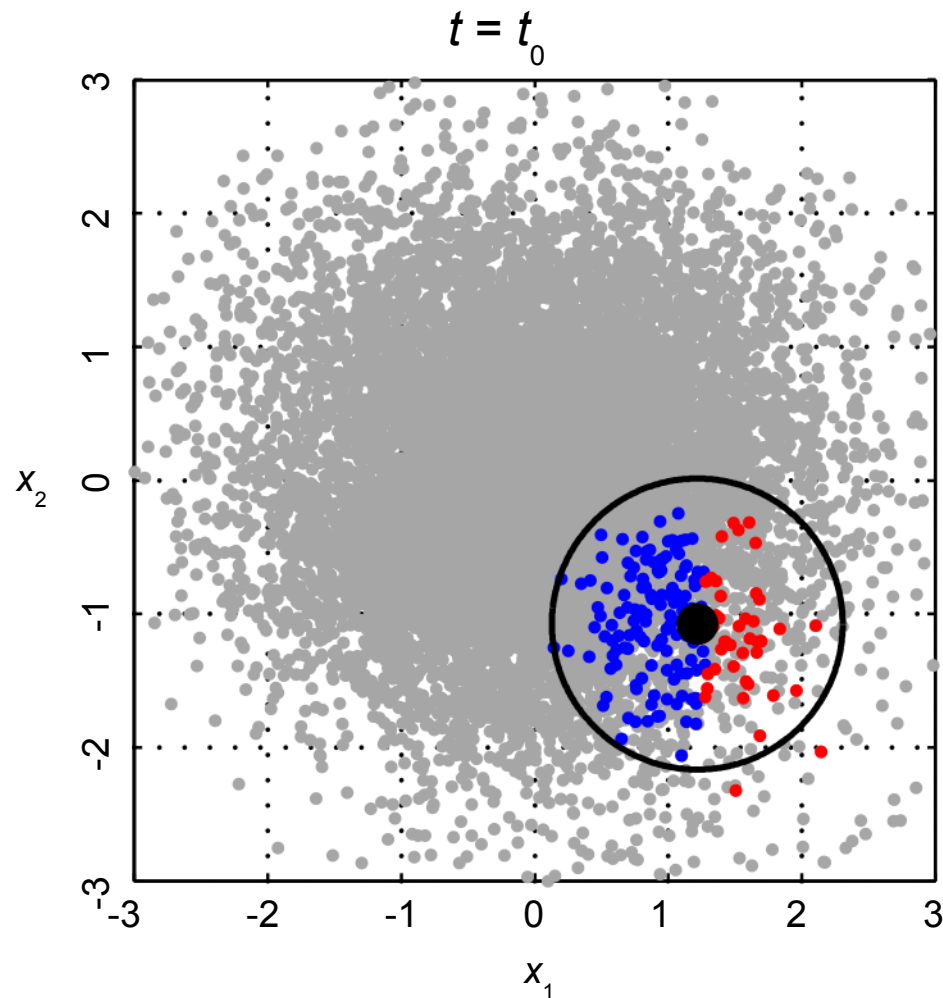
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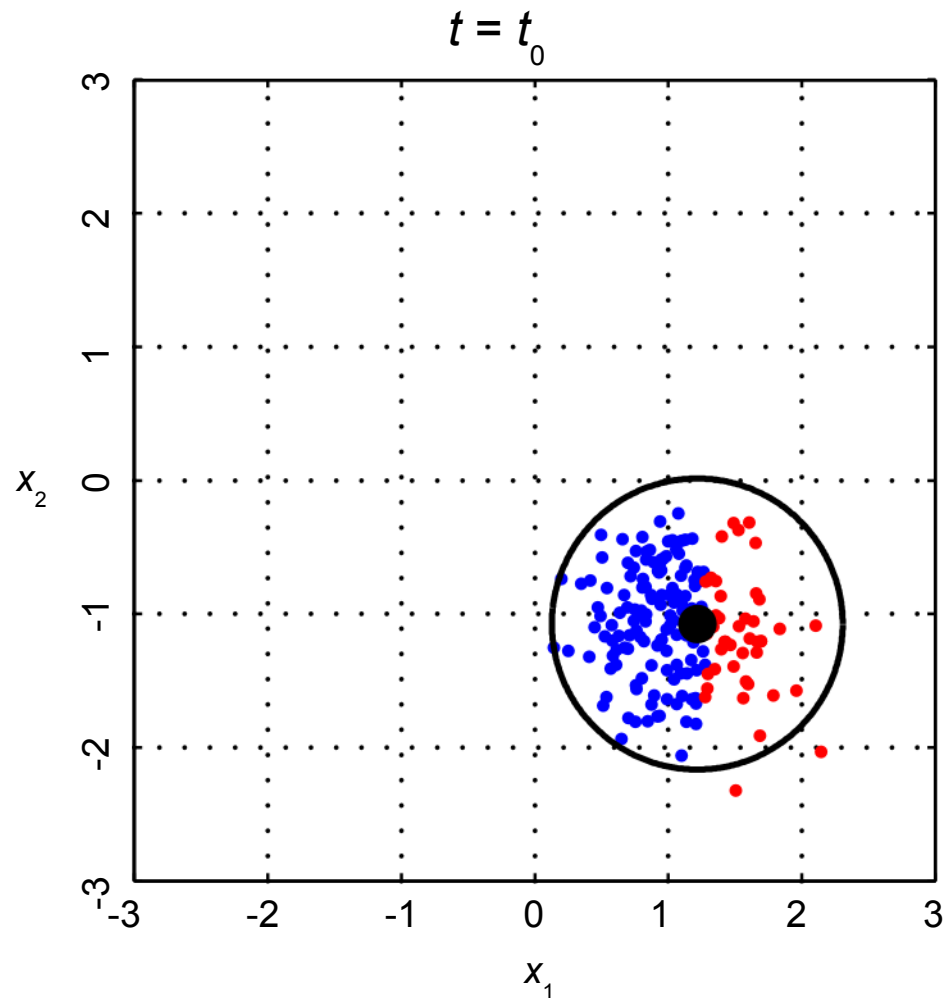
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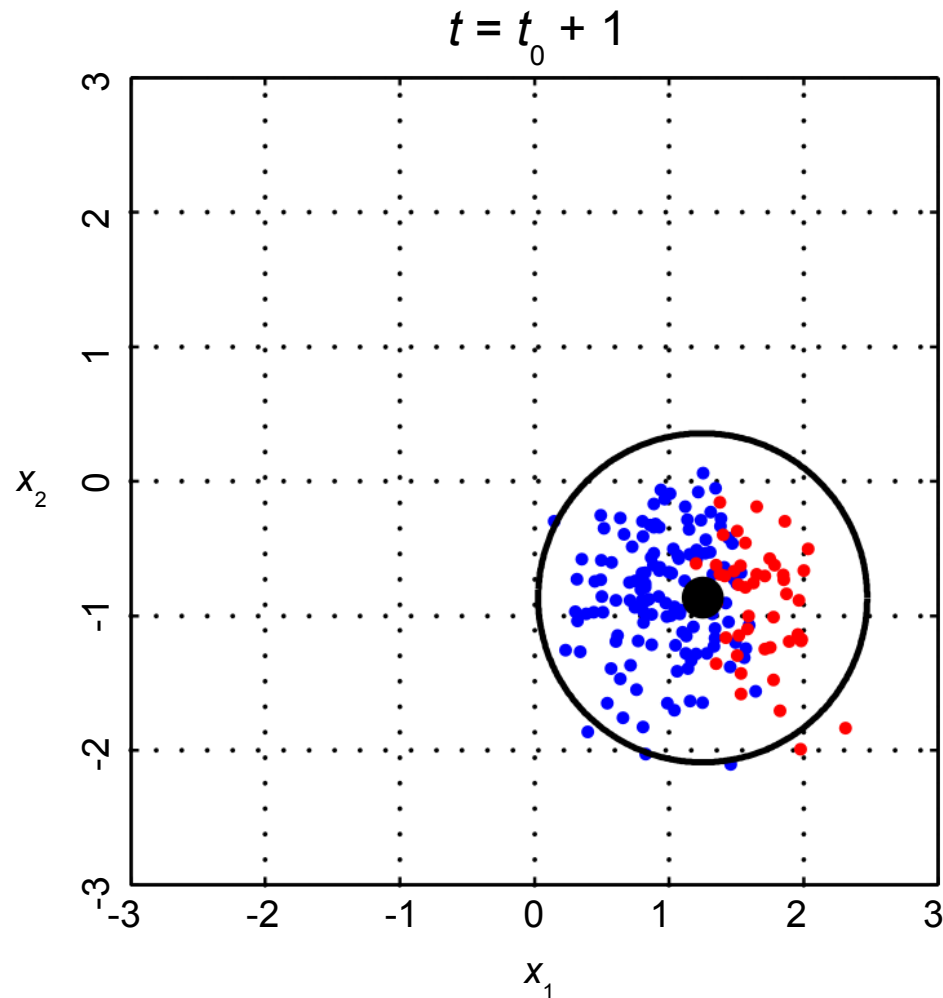
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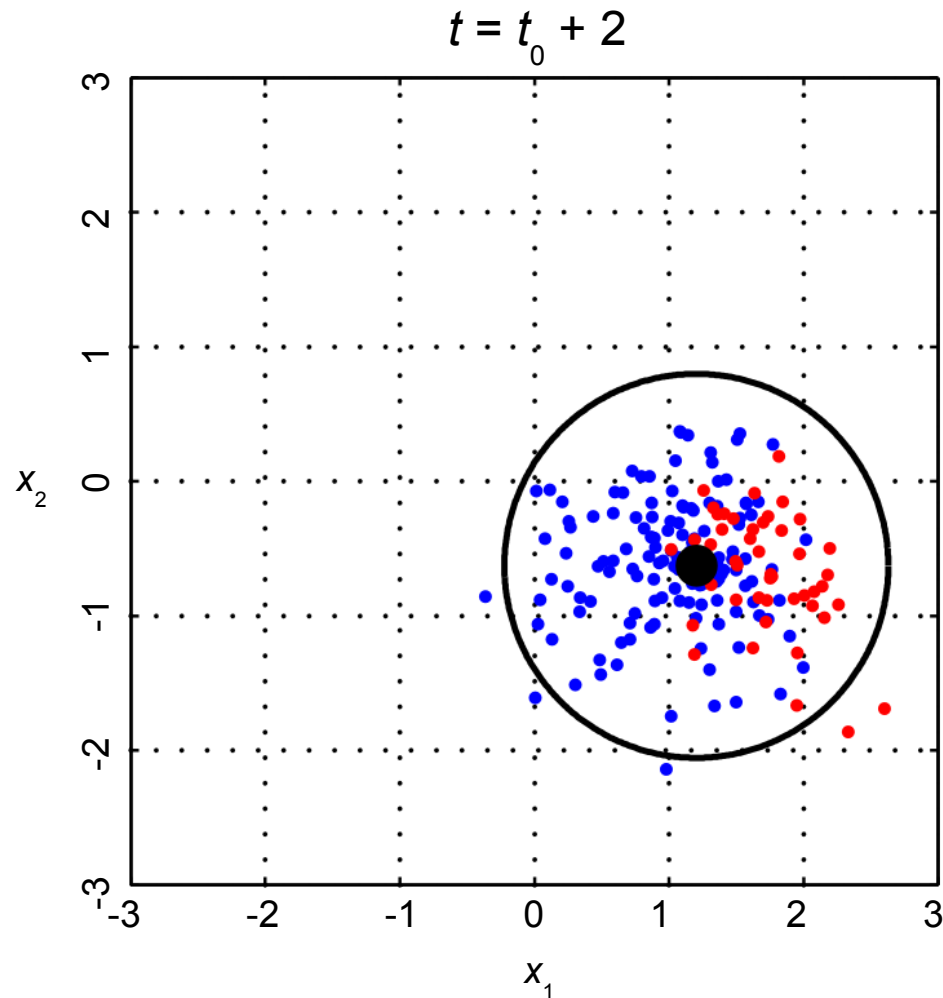
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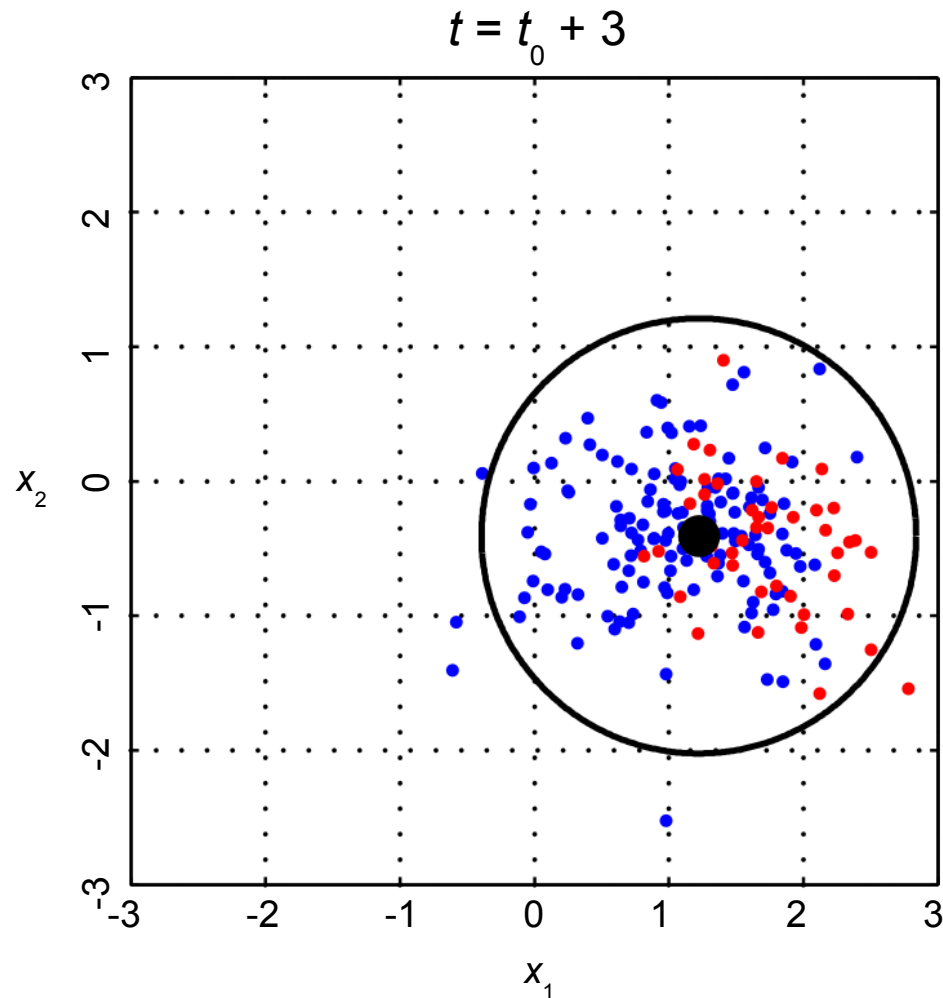
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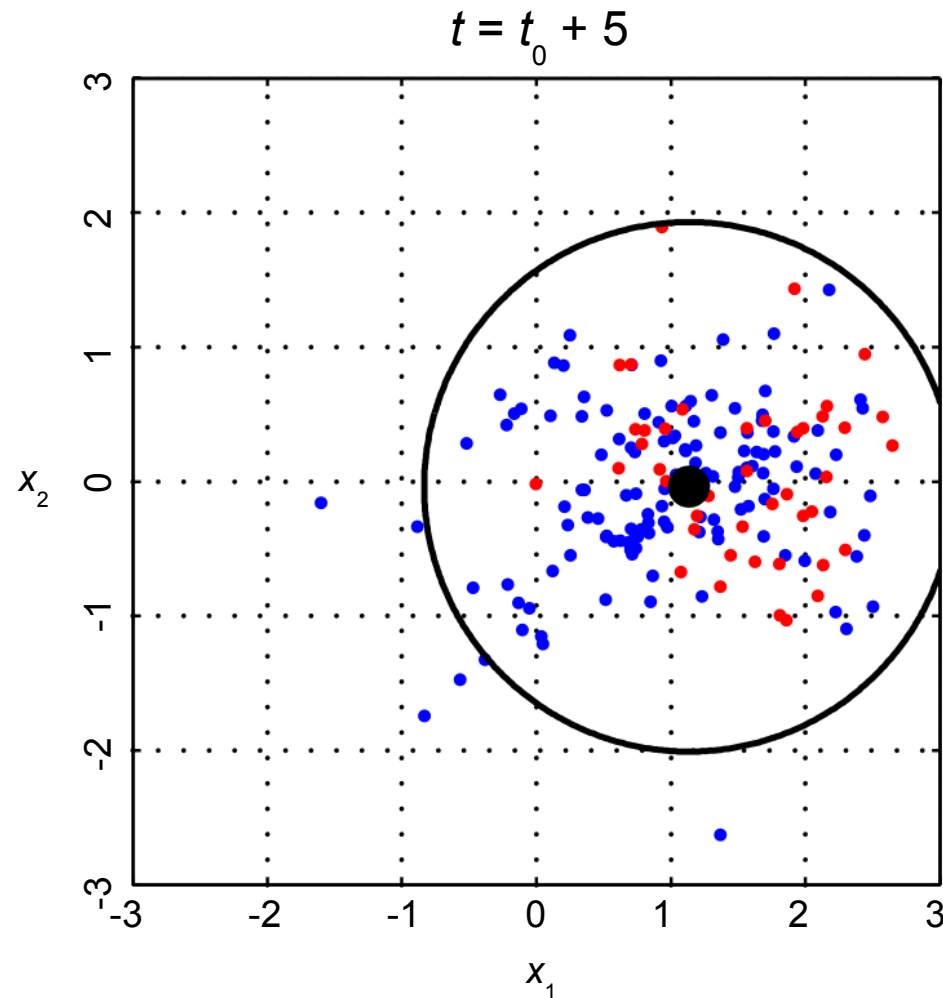
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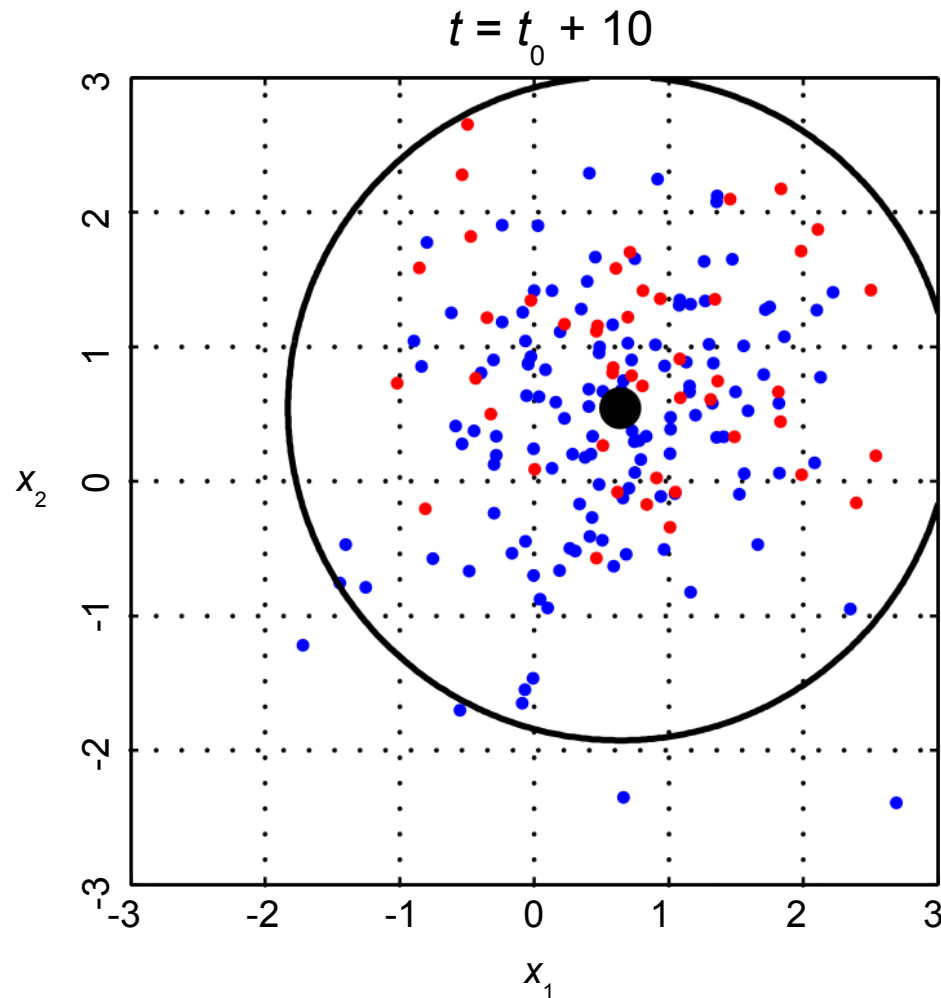
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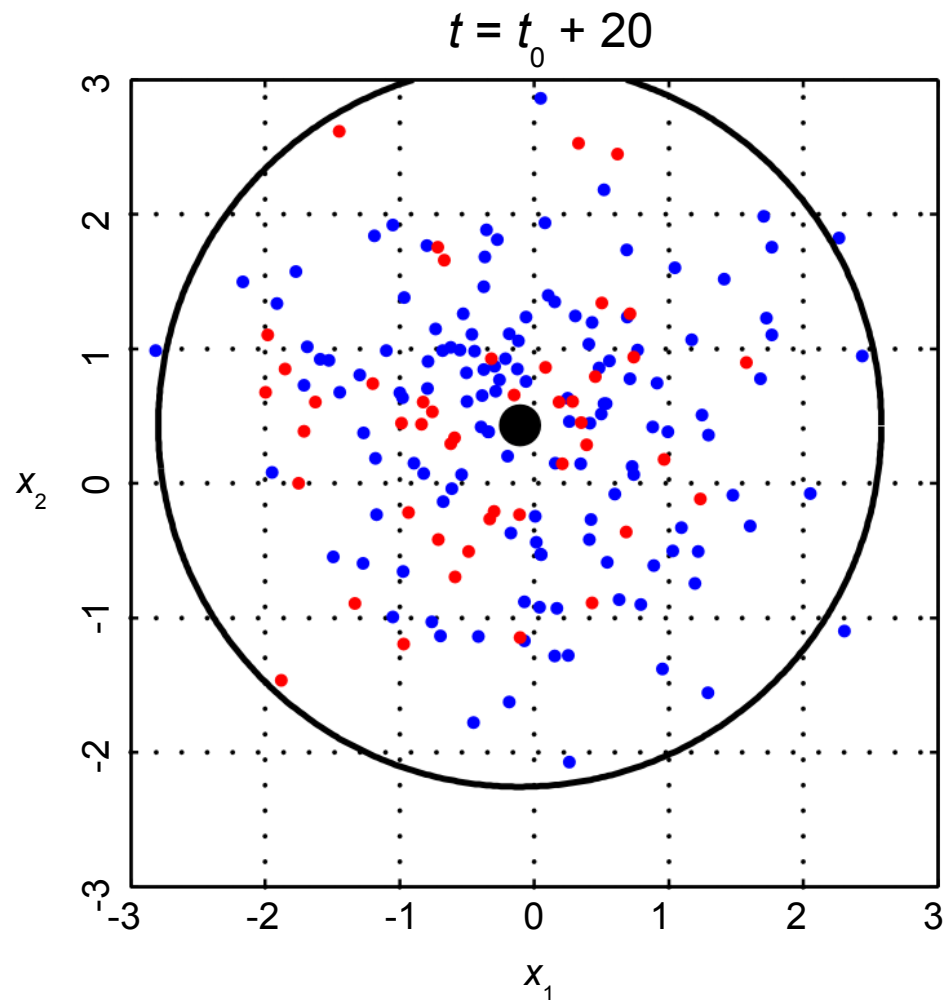
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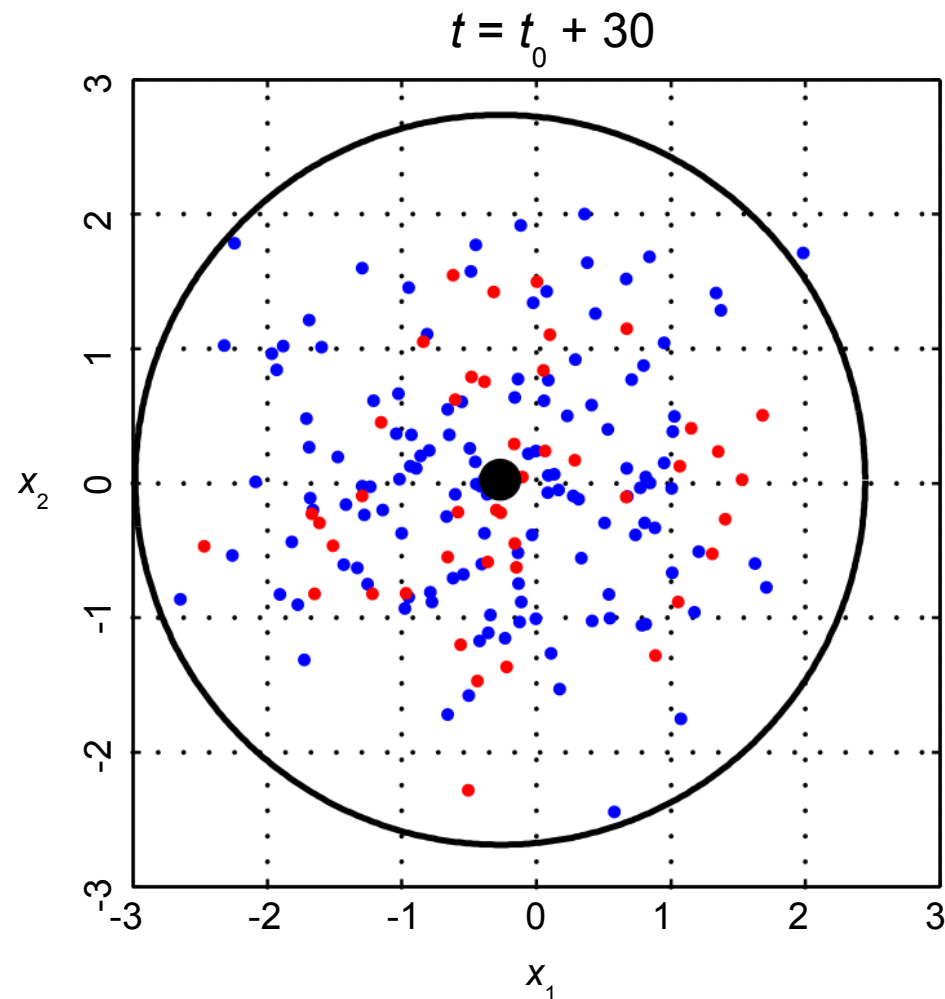
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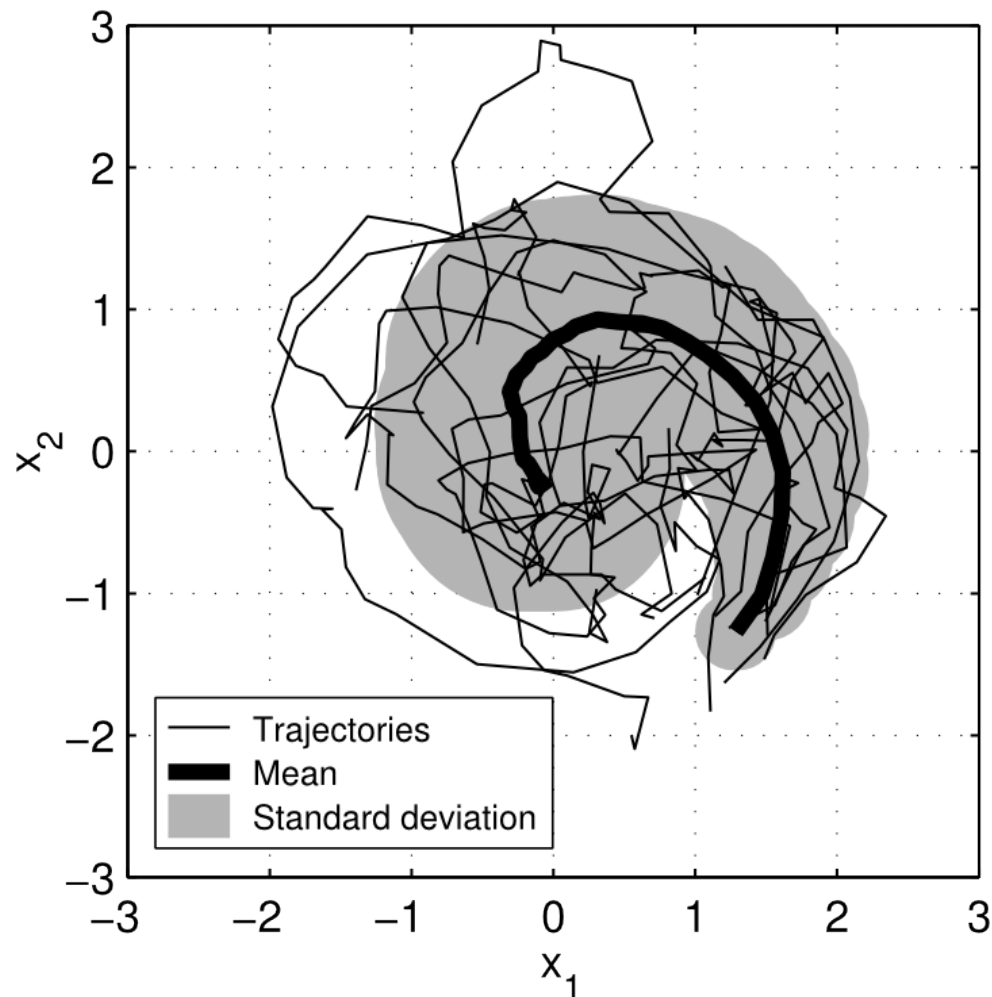
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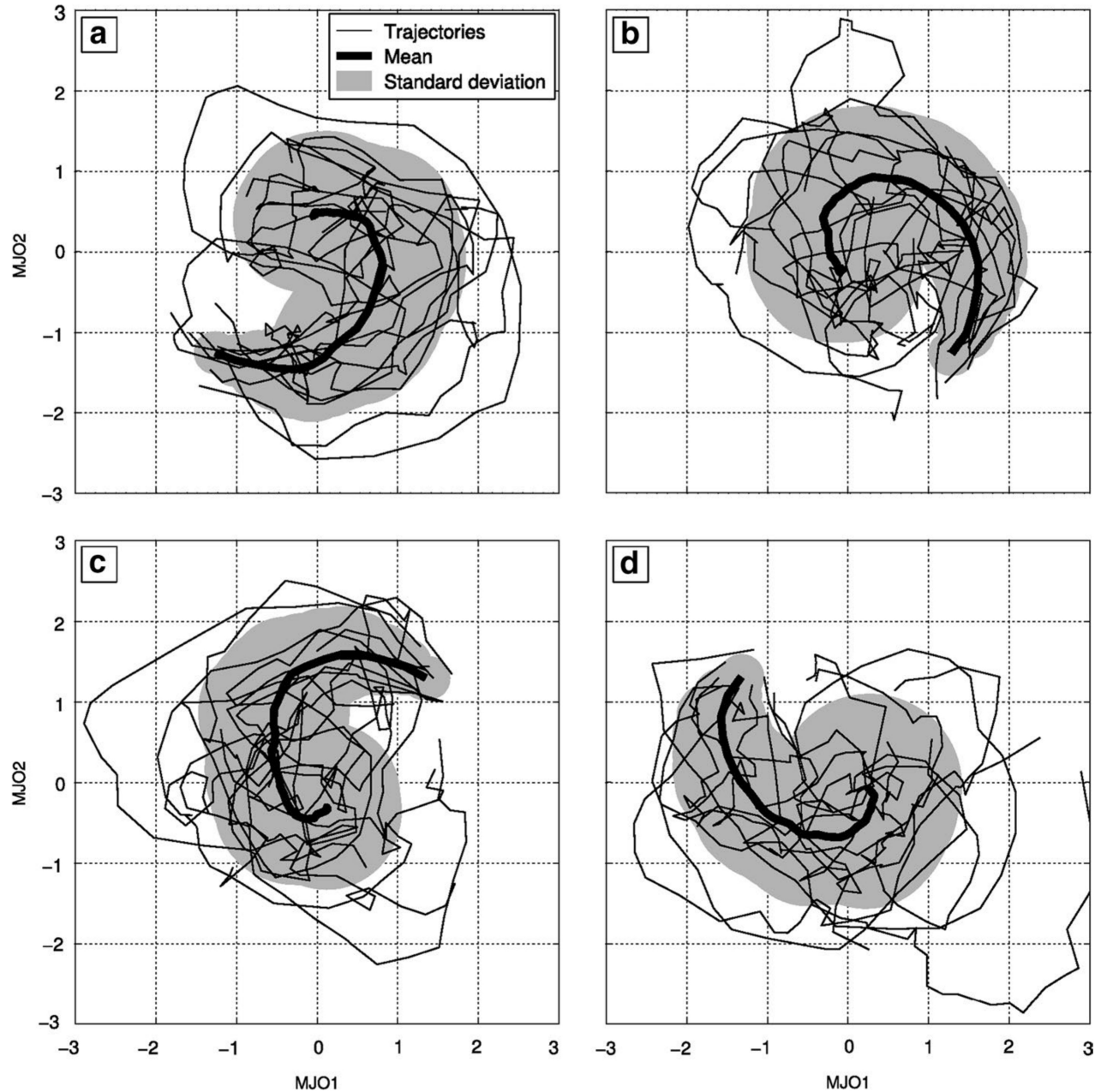
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- Distribution of all MJO index values distributed approx. as bivariate normal (grey)
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- Time evolution shows spiralling of mean, increase of spread, and ensemble mixing



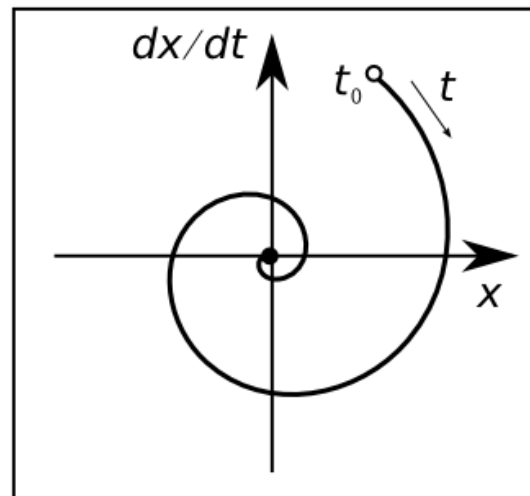
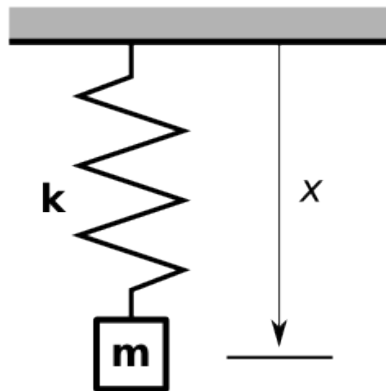
- Similar patterns for initial conditions in other phases



- Distribution of all MJO index values distributed approx. as bivariate normal (grey)

Behaviour of a damped harmonic oscillator

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = 0$$



- A damped harmonic oscillator can be expressed as a **bivariate autoregressive** (1st-order; AR1) process:

$$\mathbf{x}_{t+1} = \mathbf{A}_1 \mathbf{x}_t + \mathbf{f}_{t+1} \quad \mathbf{A}_1 = \gamma_1 \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

where $\mathbf{x} = [x_1 \ x_2]^T$ is the bivariate state vector, and $0 < \gamma_1 < 1$.

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- Forcing \mathbf{f}_t is bivariate stochastic forcing (with memory)
 - AR1 with parameter $0 < \gamma_2 < 1$ (**Model 2**)

$$\mathbf{f}_{t+1} = \mathbf{A}_2 \mathbf{f}_t + \boldsymbol{\epsilon}_{t+1} \quad \mathbf{A}_2 = \gamma_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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- Can re-express the whole system as a 4D AR1 process with $\mathbf{x} = [x_1 \ x_2 \ f_1 \ f_2]^T$ and can get expressions for **mean ($\boldsymbol{\mu}$), covariance ($\boldsymbol{\Sigma}$), and correlation ($\boldsymbol{\rho}$; mixing)**

$$\begin{aligned} \boldsymbol{\mu}_{t_0+k} &= \mathbf{A}^k \boldsymbol{\mu}_{t_0} \\ \boldsymbol{\Sigma}_{t_0+k,t_0} &= \mathbf{A}^k \boldsymbol{\Sigma}_{t_0,t_0} \end{aligned} \quad \rho_k^2 = \frac{\text{tr}(\boldsymbol{\Sigma}_{t_0+k,t_0} \boldsymbol{\Sigma}_{t_0,t_0}^{-1} \boldsymbol{\Sigma}_{t_0,t_0+k})}{\text{tr}(\boldsymbol{\Sigma}_{t_0+k,t_0+k})} \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{I} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix}$$

- A damped harmonic oscillator can be expressed as a **bivariate autoregressive** (1st-order; AR1) process:

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Parameters (time scales)

$$P = 2\pi/\theta$$

$$\tau_1 = -1/\log \gamma_1$$

$$\tau_2 = -1/\log \gamma_2$$

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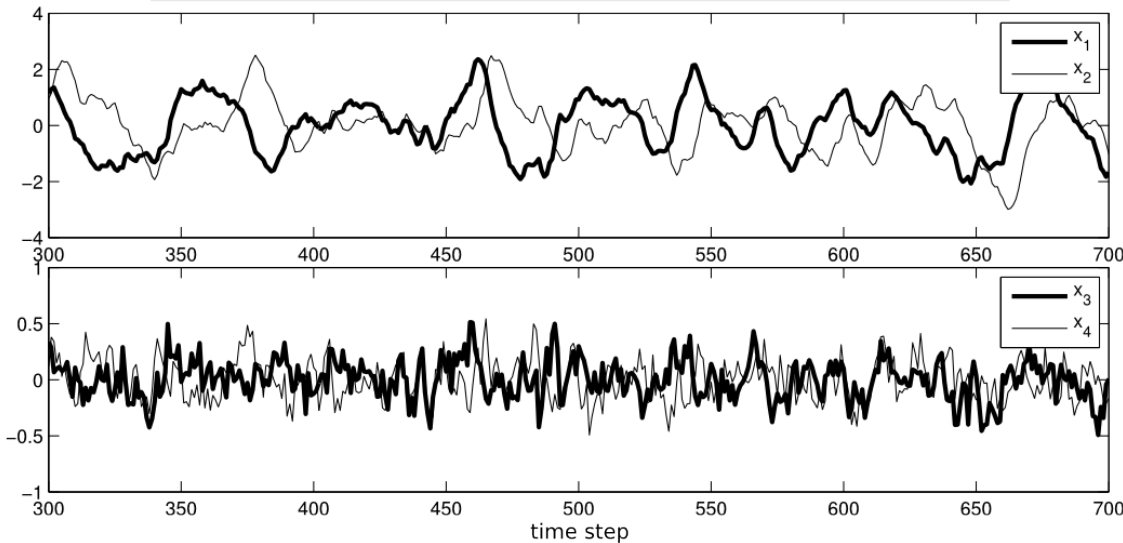
$$\boldsymbol{\Sigma}_{t_0+k,t_0} = \mathbf{A}^k \boldsymbol{\Sigma}_{t_0,t_0}$$

$$\rho_k^2 = \frac{\text{tr}(\boldsymbol{\Sigma}_{t_0+k,t_0} \boldsymbol{\Sigma}_{t_0,t_0}^{-1} \boldsymbol{\Sigma}_{t_0,t_0+k})}{\text{tr}(\boldsymbol{\Sigma}_{t_0+k,t_0+k})}$$

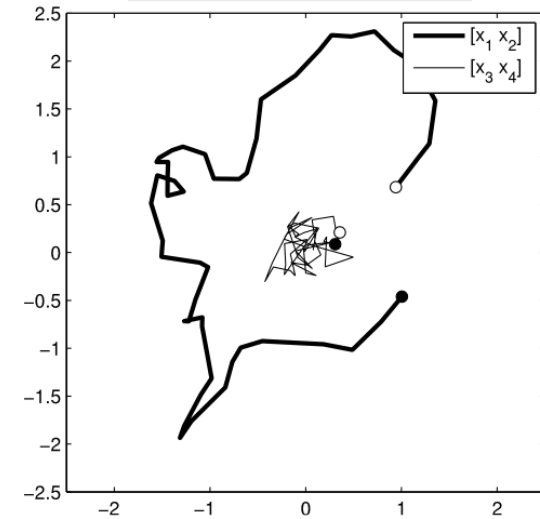
$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{I} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix}$$

- Model behaviour with $P = 50$ days, $\tau_1 = 15$ days, $\tau_2 = 2$ days

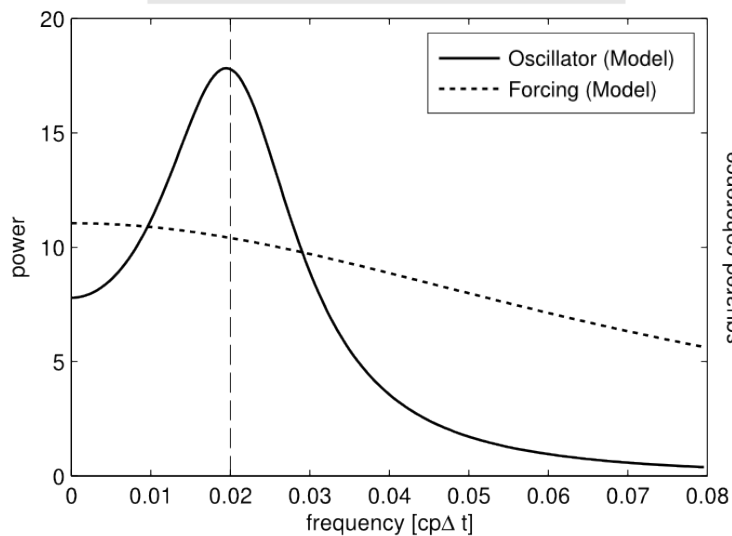
Oscillator model: Components 1 (—) and 2 (—)



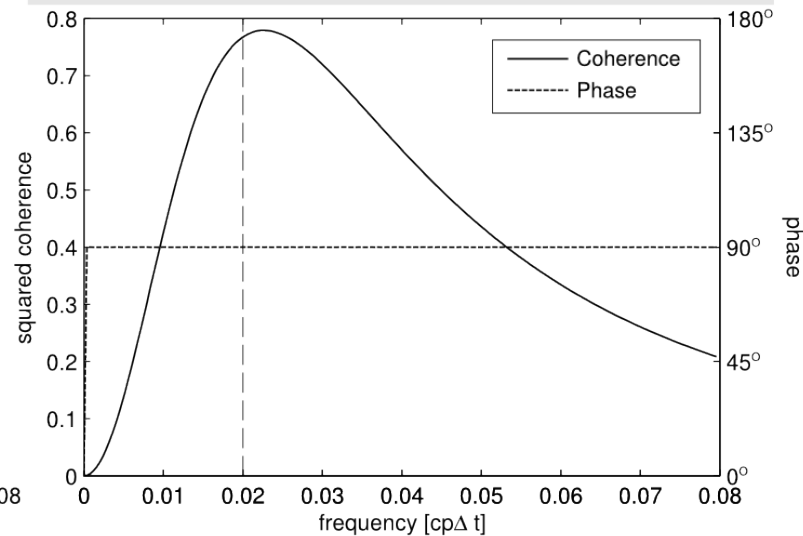
Phase Space



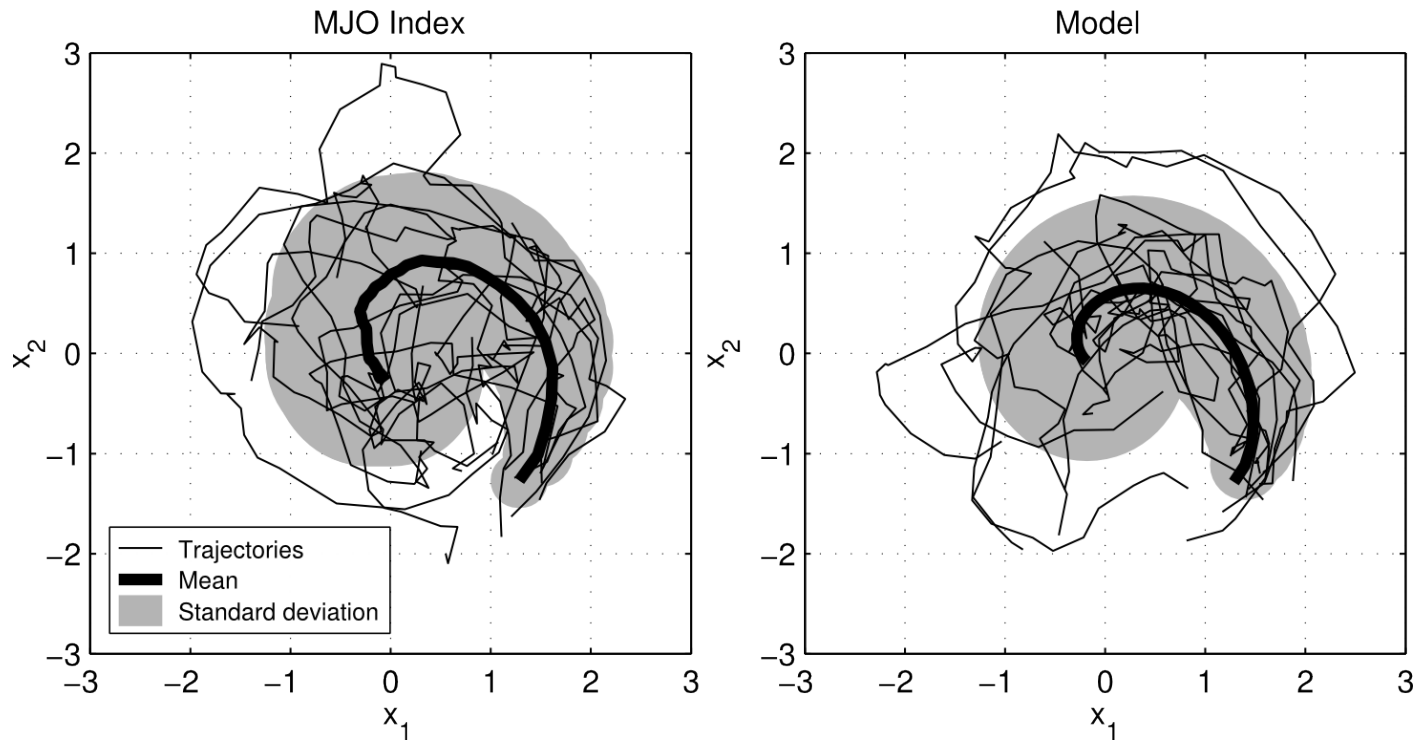
Oscillator power spectra



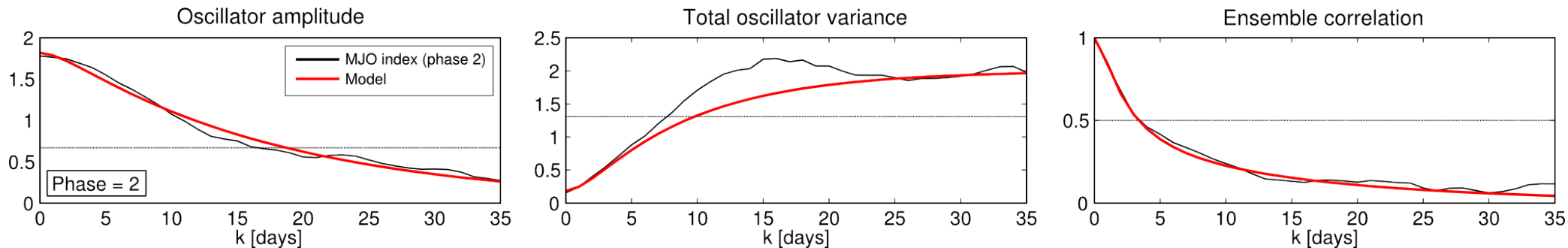
Oscillator coherence and phase spectra



- Can use model to get ensemble statistics...

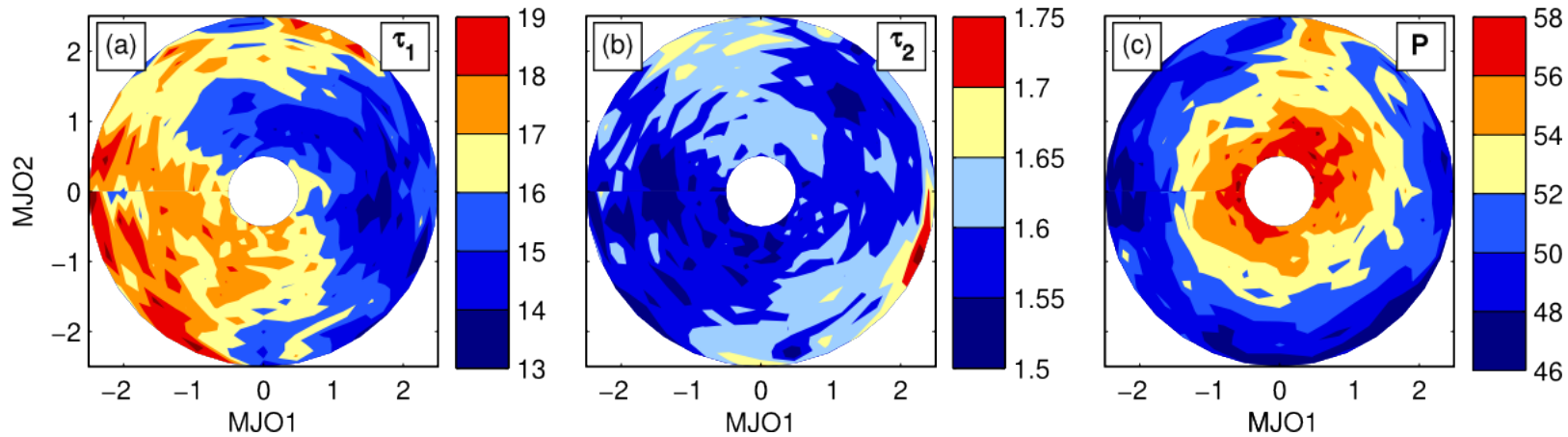


- ...and three time scales (mean, variance, correlation)

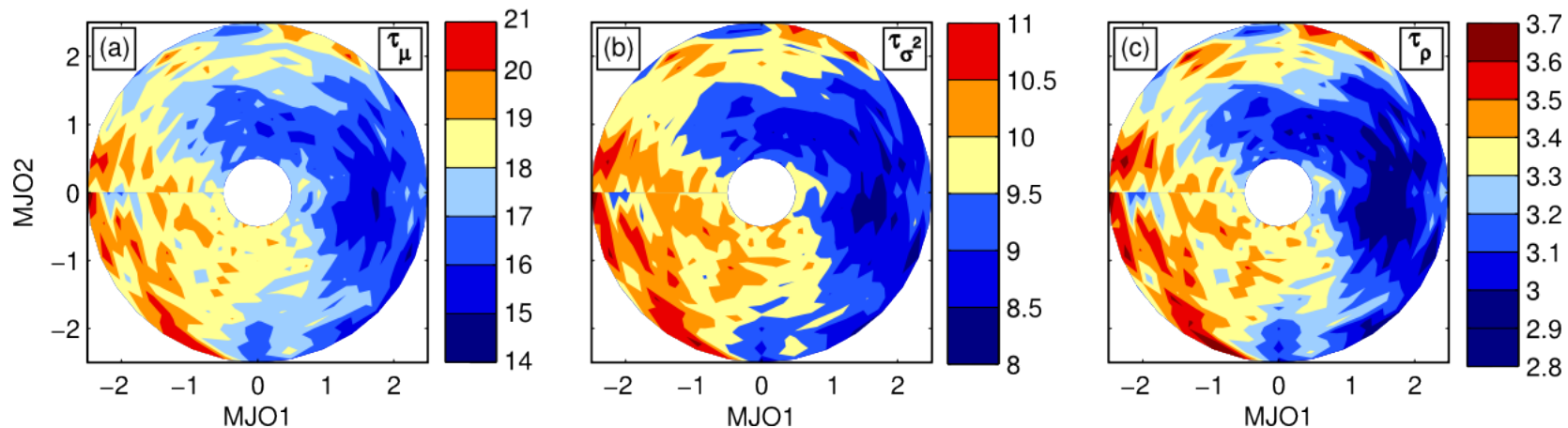


- Model can be fit (using max. likelihood estimation) to ensembles of trajectories

- Model fit for initial conditions covering most of phase space, to map out predictability
- Model parameters:



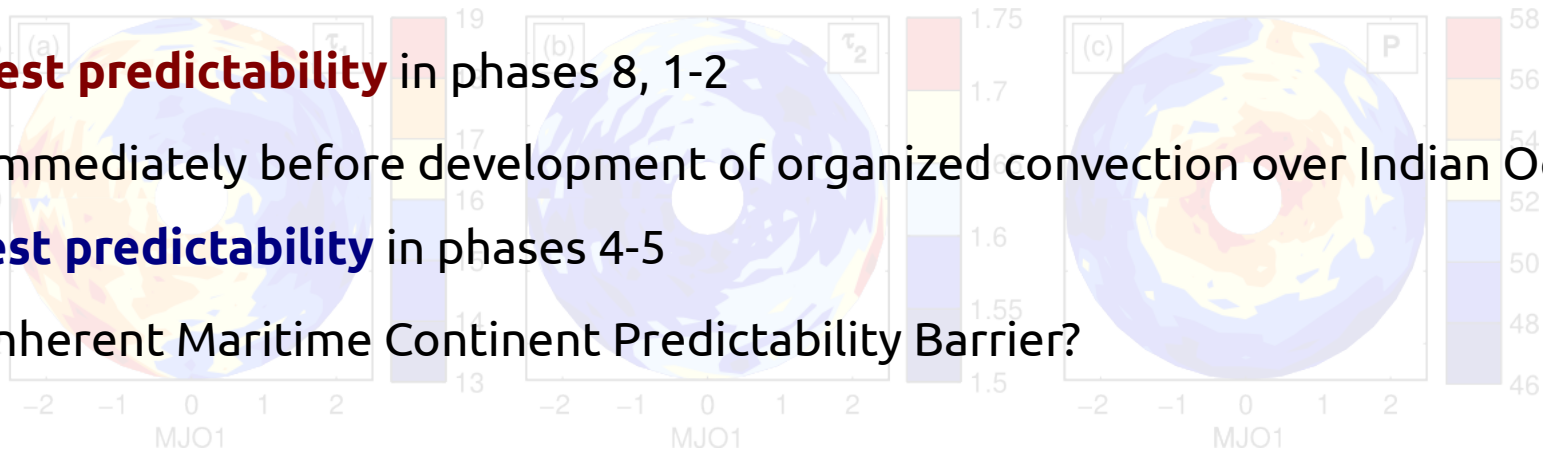
- Predictability time scales:



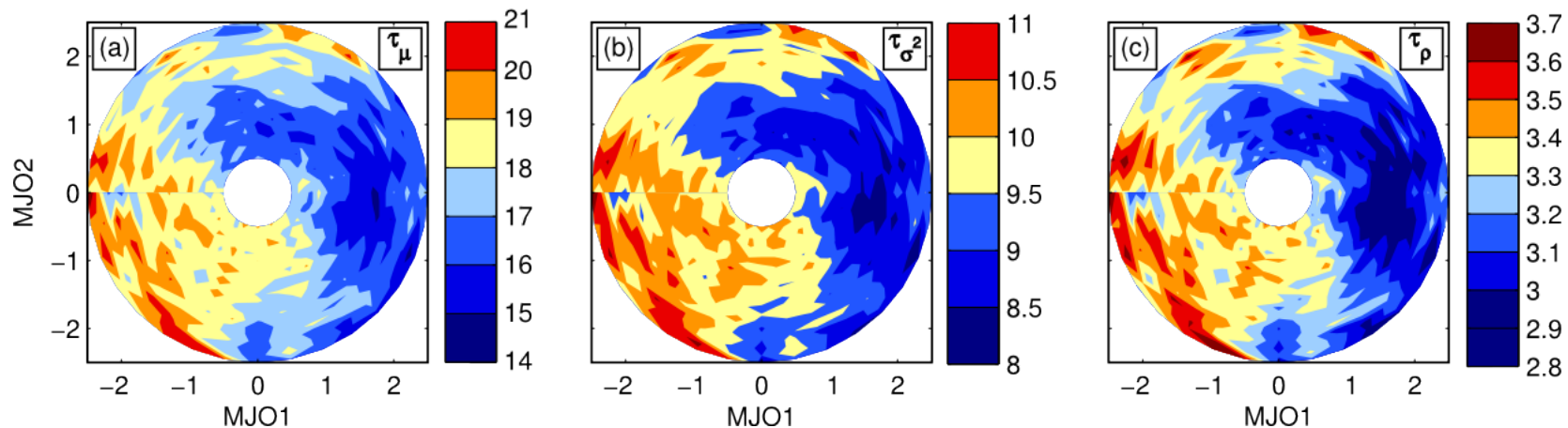
- Yin-Yang pattern: Strong dependency on initial condition

- Model fit for initial conditions covering most of phase space, to map out predictability
- Model parameters:

- **Highest predictability** in phases 8, 1-2
 - Immediately before development of organized convection over Indian Ocean
- **Lowest predictability** in phases 4-5
 - Inherent Maritime Continent Predictability Barrier?

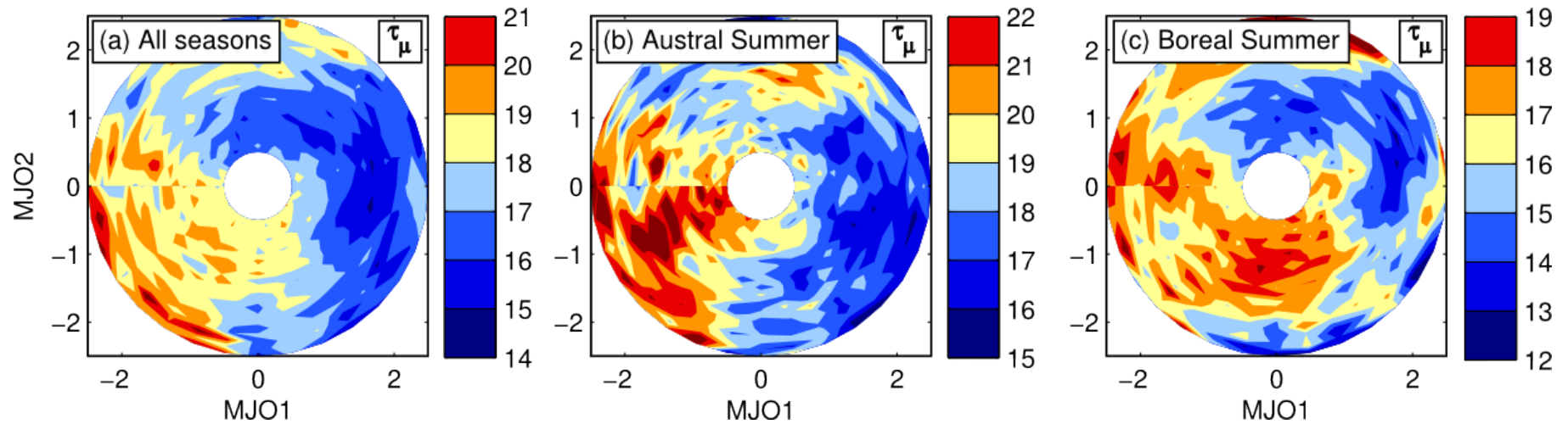


- Predictability time scales:



- Yin-Yang pattern: Strong dependency on initial condition

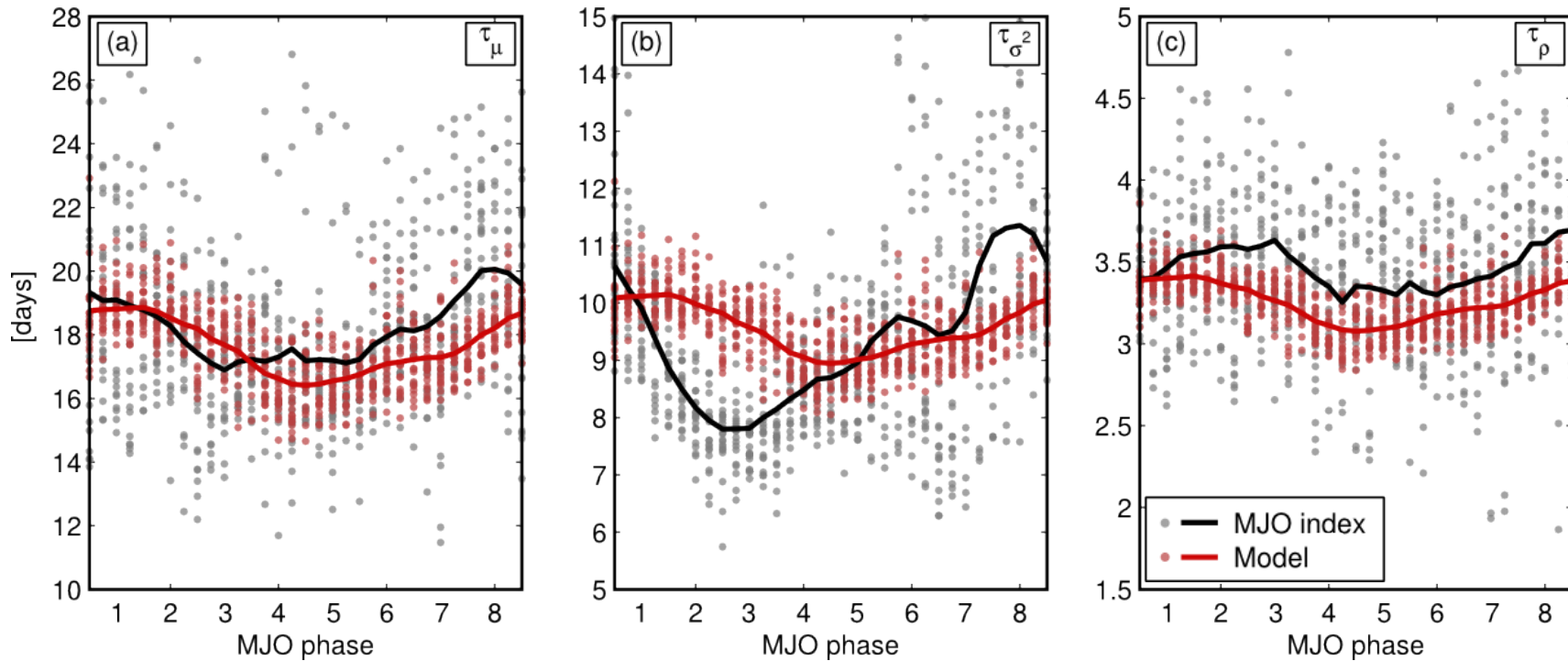
- Seasonality of predictability time scales



- Highest** in Austral Summer (when MJO is strongest)
- Phase-timing of peak predictability **rotates** by ~ 1 phase with season!
- Note: MJO index does not measure *Boreal Summer Intraseasonal Oscillation*

- MJO behaviour reminiscent of damped harmonic oscillator
- Simple model captures temporal and spectral properties of MJO as well as basic predictability
 - **2-3 week predictability for ensemble mean**
 - **~3 days for within ensemble correlation**
- MJO predictability varies with
 - Initial MJO phase
 - Highest in phases preceding MJO development over Indian Ocean
 - Lowest in phases initialized over Maritime Continent → Maritime Continent Prediction Barrier may be a property of the MJO itself and not a failing of numerical models
 - Season
 - Highest in Austral Summer (when MJO is strongest)
- **Publications**
 - Model first introduced: Oliver, E. C. J. and K. R. Thompson (2012), *A Reconstruction of Madden-Julian Oscillation Variability from 1905 to 2008*, **Journal of Climate**, 25(6)
 - More detail, including dependency on phase/season: Oliver, E. C. J. and K. R. Thompson (2015), *Predictability of the Madden Julian Oscillation index: Seasonality and dependence on MJO phase*, **Climate Dynamics** (online)

- Phase dependency of predictability time scales



- Highest predictability** in phases 8, 1-2
 - Immediately before development of organized convection over Indian Ocean
- Lowest predictability** in phases 6-7
 - Inherent Maritime Continent Predictability Barrier?